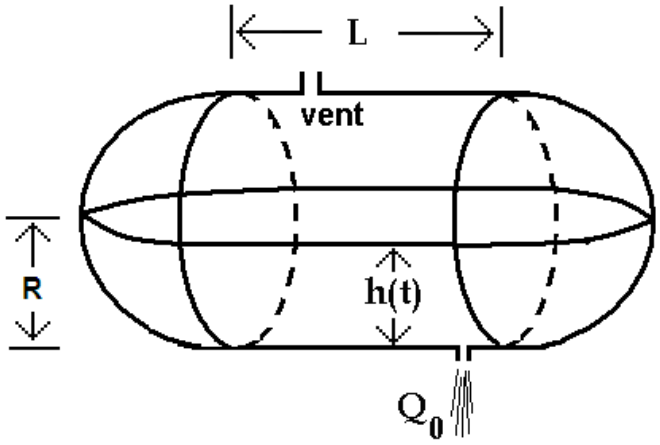
Draining of a storage tank

The storage tank shown below is used to store and supply water. The tank is cylindrical in shape and has hemispherical ends.



The tank radius is 1 m and the cylindrical section is L = 2 m in length.

We wish to design the outlet so that the volumetric flow rate Q_o = 0.001 m^3/s when the tank is half full.

Find the orifice areas A_0 (in m^2) that meets this objective and then determine how long it will take the tank to completely drain if it starts full.

The flow rate out of the tank is given by a modification of Torricelli's equation:

$$Q_0 = A_0 C_d \sqrt{2gh}$$

where A_0 is the orifice area, h is the liquid level height, C_d is the discharge coefficient (C_d=0.6) and g is the gravitational constant (g=9.8 m/s^2)

A mathematical model for the tank, based on conservation of mass, is

$$\frac{d(\rho V)}{dt} = -\rho Q_0$$

where ρ is the density of water and V is the volume of water in the tank at time t.